We explore the dynamics of network motifs, illustrated in Fig. 1 and provided as ordinary differential equations (ODEs) in the matching questions. For an on-step, the input signal $S(t) = 0$ for $t < 0$ and $S(t) > 0$ for $t \geq 0$. For an off-step, $S(t) > 0$ for $t < 0$ and $S(t) = 0$ for $t \geq 0$. At time 0, the system has steady-state values consistent with the input for $t < 0$.

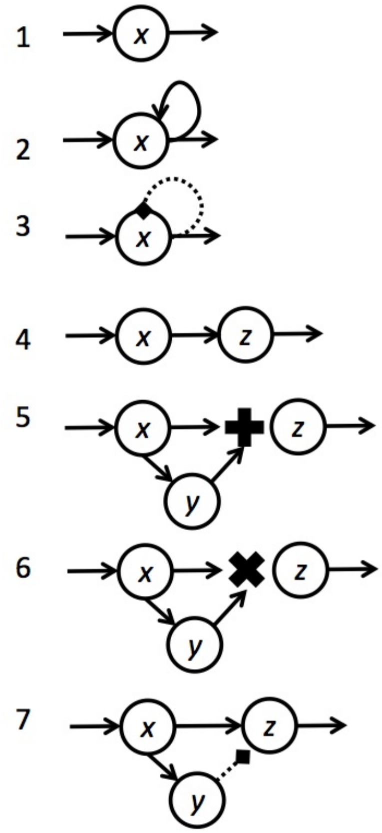

Figure 1: Network motifs illustrate ODEs in the problems below. Solid lines with arrows indicate positive regulation; dotted lines with blocks indicate negative regulation.

1. The standard two-state switch has ODE

$$\dot{x}(t) = \beta\Theta[S(t) > 0] - \alpha x(t).$$

   For an on-step, provide the response $x(t)$ and the time $t_{1/2}$ when the response is half complete.

2. With weak positive feedback, the ODE is

$$\dot{x}(t) = \beta\Theta[S(t) > 0] + \beta_f x(t) - \alpha x(t).$$

(a) Consider the on-step. Provide the response $x(t)$ and the time $t_{1/2}$ when the response is half complete. Does positive feedback increase or decrease the response time? Does positive feedback increase or decrease the final steady-state value $\lim_{t\to\infty} x(t)$?

(b) At what value of the positive feedback parameter β_f does $t_{1/2}$ become non-physical (for example, infinite or negative)?

(c) With strong positive feedback, a better ODE is

$$\dot{x}(t) = \beta\Theta[S(t) > 0] + \beta_f\Theta[x(t) > K] - \alpha x(t).$$

Consider an input signal with duration τ: $S(t) = 0$ for $t < 0$, $S(t) > 0$ for $0 \le t < \tau$, and $S(t) = 0$ for $t > \tau$. What are the smallest values of β, β_f, and τ that permit the system to support its own production as $t \to \infty$?

3. With negative autoregulation, the ODE is

$$\dot{x}(t) = \beta\Theta[S(t) > 0]\Theta[x(t) < K] - \alpha x(t),$$

and assume that $\beta/\alpha > K$. Provide the response $x(t)$ for $S(t) = 0$ for $t < 0$ and $S(t) > 0$ for $t > 0$. Calculate the time $t_{1/2}$ when the response is half complete. Is the response faster or slower than the system without negative feedback, Question 1?

4. A basic cascade has ODE

$$\begin{aligned}\dot{x}(t) &= \beta\Theta[S(t) > 0] - \alpha x(t)\\ \dot{z}(t) &= \beta\Theta[x(t) > K] - \alpha z(t).\end{aligned}$$

Assume that $\beta/\alpha > K$, and note that y is skipped; it will be added later as an intermediate component of the cascade.

(a) For the on-step, provide $x(t)$, $z(t)$. Provide $t_{1/2}$ for x and z.

(b) For the off-step, provide $x(t)$, $z(t)$. Provide $t_{1/2}$ for x and z.

5. Protein y is also a transcription factor for protein z. If either x or y can activate z, for example by binding as homotetramers, the motif functions as an OR gate with ODE

$$\begin{aligned}\dot{x}(t) &= \beta\Theta[S(t) > 0] - \alpha x(t)\\ \dot{y}(t) &= \beta\Theta[x(t) > K] - \alpha y(t)\\ \dot{z}(t) &= \beta\Theta[(x(t) > K) \text{ or } (y(t) > K)] - \alpha z(t).\end{aligned}$$

Provide $t_{1/2}$ for z for the on-step and the off-step. How do these differ from the simpler cascade, Question 4?

6. If x and y bind as a heteromeric complex to activate z, the motif functions as an AND gate with ODE

$$\begin{aligned}\dot{x}(t) &= \beta\Theta[S(t) > 0] - \alpha x(t) \\ \dot{y}(t) &= \beta\Theta[x(t) > K] - \alpha y(t) \\ \dot{z}(t) &= \beta\Theta[(x(t) > K) \text{ and } (y(t) > K)] - \alpha z(t).\end{aligned}$$

   Provide $t_{1/2}$ for z for the on-step and the off-step. How do these differ from the simpler cascade, Question 4?

7. If the intermediate transcription factor y bind as a repressor, the motif functions as a derivative operator or edge detector with ODE

$$\begin{aligned}\dot{x}(t) &= \beta\Theta[S(t) > 0] - \alpha x(t) \\ \dot{y}(t) &= \beta\Theta[x(t) > K] - \alpha y(t) \\ \dot{z}(t) &= \beta\Theta[(x(t) > K) \text{ and } (y(t) < K)] - \alpha z(t).\end{aligned}$$

   For an on-step input, provide the output $z(t)$, the time $t_{\max}$ when z has its maximum value, and the maximum value $z_{\max} = z(t_{\max})$. Provide the two times when $z(t) = z_{\max}/2$.