1. Evaluate $\oint dz \frac{1}{z}$ for the following closed contours expressed in polar coordinates, $z = re^{i\theta}$ with $r > 0$.
   (a) A single counter-clockwise loop, $\theta = 0$ to 2π.
   (b) A single clockwise loop, $\theta = 2\pi$ to 0.
   (c) A double counter-clockwise loop, $\theta = 0$ to 4π.
   (d) A complicated path with m counter-clockwise loops and n clockwise loops.

2. Evaluate
$$\oint \frac{ds}{2\pi i} \frac{e^{st}}{(s+a)(s+b)}$$
over the following closed contours, each a single counter-clockwise loop, with $a > b > 0$.
   (a) A contour including $-a$ but excluding $-b$.
   (b) A contour including $-b$ but excluding $-a$.
   (c) A contour up the imaginary axis and then closed in the left half-plane: $z = -i\infty \to +i\infty \to -\infty + i\infty \to -\infty - i\infty \to -i\infty$.
   (d) A contour up the imaginary axis and then closed in the right half-plane: $z = -i\infty \to +i\infty \to \infty + i\infty \to \infty - i\infty \to -i\infty$.
   (e) Not a closed contour but just a path up the imaginary axis, $z = -i\infty \to +i\infty$.

3. Consider the integral
$$\int_{-i\infty}^{i\infty} \frac{ds}{2\pi i} \frac{e^{st}}{(s+a)^2}$$
with $a > 0$.
   (a) Evaluate the integral using contour integrals by expanding e^{st} in a Taylor series around $s = -a$, $e^{st} = e^{-at} e^{(s+a)t} = e^{-at}[1 + (s+a)t + (s+a)^2 t^2/2 + (s+a)^3 t^3/3! + \ldots]$.
   (b) Evaluate the integral by first evaluating
$$\int_{-i\infty}^{i\infty} \frac{ds}{2\pi i} \frac{e^{st}}{(s+a)(s+b)}$$
   and then taking the limit $b \to a$.
   (c) Evaluate
$$\int_{-i\infty}^{i\infty} \frac{ds}{2\pi i} \frac{e^{st}}{s^2}.$$

4. Contour integrals simplify Fourier transforms. Let
$$f(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{e^{i\omega t}}{\omega^2 + a^2}$$
with $a > 0$.

(a) Where are the poles?

(b) For $t > 0$, what is the value of the integrand for $\omega \to +i\infty$ and for $\omega \to -i\infty$? Can you close the contour in the upper half-plane or the lower half-plane?

(c) What is $f(t)$ for $t > 0$?

(d) For $t < 0$, in which half-plane should the contour be closed? What is $f(t)$ for $t < 0$?

(e) Noting that $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$, what is

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\cos(\omega t)}{\omega^2 + a^2}?$$

What is

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega t)}{\omega^2 + a^2}?$$