In this homework, $\mathscr{L}$ stands for the Laplace transform, $\tilde{f}(s) = \mathscr{L}[f(t)] = \int_0^\infty dt e^{-st} f(t)$.

1. Simple Laplace transforms. Provide $\tilde{f}(s)$ for the following $f(t)$.
   (a) $f(t) = 1$
   (b) $f(t) = 0$ if $t < a$, $f(t) = 1$ if $t \geq a$, and $a \geq 0$.
   (c) $f(t) = e^{-at}$
   (d) $f(t) = t$
2. Laplace transforms of powers through generating functions.
   (a) Let $f(t) = \lim_{a \to 0}(-d/da)e^{-at}$. What is $\tilde{f}(s)$?
   (b) Let $\tilde{g}(s,a) = -(d/da)\mathscr{L}[e^{-at}]$. What is $\tilde{g}(s,a)$?
   (c) If functions are well-behaved, then $\tilde{f}(s)$ should equal $\lim_{a \to 0} \tilde{g}(s,a)$. Calculate $\tilde{f}(s)$ with this method.
   (d) Describe how you could use this approach to calculate $\mathscr{L}(t^n)$ for $n = 1, 2, 3, \ldots$.
3. Convolution. Let $\tilde{f}(s) = 1/(s+a)$ and $\tilde{g}(s) = 1/(s+b)$.
   (a) What are $f(t)$ and $g(t)$?
   (b) What is $\int_0^\infty dt \int_0^t dt' e^{-st} f(t-t')g(t')$?